## In a nutshell: Gradient descent

Given a continuous and differentiable real-valued function $f$ of a vector variable with one initial approximation of a minimum $\mathbf{u}_{0}$, gradient descent converts the problem of approximating a solution in $n$ dimensions to approximating the solution in one dimension, one step at a time.

Parameters:
$\varepsilon_{\text {step }} \quad$ The maximum error in the value of the minimum cannot exceed this value.
$\varepsilon_{\mathrm{abs}} \quad$ The difference in the value of the function after successive steps cannot exceed this value.
$N \quad$ The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k>N$, we have iterated $N$ times, so stop and return signalling a failure to converge.
3. Calculate $\nabla f\left(\mathbf{u}_{k}\right)$ and define a function $f\left(\mathbf{u}_{k}-\alpha \nabla f\left(\mathbf{u}_{k}\right)\right)$. This is a real-valued function of a real variable $\alpha$, so use an algorithm to find a minimum in the direction of $-\nabla f\left(\mathbf{u}_{k}\right)$. Let this minimum be $\alpha_{k}>0$.
4. Set $\mathbf{u}_{k+1} \leftarrow \mathbf{u}_{k}-\alpha_{k} \nabla f\left(\mathbf{u}_{k}\right)$.
5. If $\left\|\mathbf{u}_{k+1}-\mathbf{u}_{k}\right\|_{2}<\varepsilon_{\text {step }}$ and $\left|f\left(\mathbf{u}_{k+1}\right)-f\left(\mathbf{u}_{k}\right)\right|<\varepsilon_{\mathrm{abs}}$, return $\mathbf{x}_{k+1}$.
6. Return to Step 2.
